

• Decimal Place Value

Power Up

Power Up I

mental math Find half of each number in **a-d**.

- a. Number Sense: 24
- b. Number Sense: 50
- c. Number Sense: 46
- d. Number Sense: 120
- **e. Money:** The apples cost \$3.67. Lindsay paid for them with a \$5 bill. How much change should she receive?
- f. Estimation: About how many feet is 298 yards? (*Hint:* Round the number of yards to the nearest hundred yards before mentally calculating.)
- g. Calculation: 6×7 , -2, +30, +5, $\div 3$
- h. Roman Numerals¹: Write 12 in Roman numerals.
- problemChoose an appropriate problem-solving strategy to solve this
problem. There were two gallons of punch for the class party.
The punch was served in 8-ounce cups. Two gallons of punch
was enough to fill how many cups? (Remember that 16 ounces
is a pint, two pints is a quart, two quarts is a half gallon, and two
half gallons is a gallon.)

¹ In Lessons 91–105, the Mental Math section "Roman Numerals" reviews concepts from Appendix Topic A. You may skip these Mental Math problems if you have not covered Appendix Topic A.



Math Language

The mill was first introduced in 1786 by the Continental Congress as a money amount worth 1/1000 of the federal dollar. Some states issued a token in this amount as a way to pay sales tax, but by 1960 the mill was no longer made. Today, the cost of gasoline is still represented in tenths of a cent. For example, \$3.019 per gallon is three dollars, one penny, and nine mills.

Thinking about money can help us understand decimal place value.



We have used \$100, \$10, and \$1 bills to represent place values to the left of the decimal point. To the right of the decimal point, we see the tenths, hundredths, and thousandths places. Since a dime is $\frac{1}{10}$ of a dollar, the tenths place is for dimes. The number of pennies goes in the hundredths place because a penny is $\frac{1}{100}$ of a dollar. The third place to the right of the decimal point is the thousandths place. We do not have a coin for a thousandth of a dollar, but we do have a name for it. A thousandth of a dollar is a *mill*, so one mill is $\frac{1}{1000}$ of a dollar. Ten mills equals one cent.

Which digit in 12.875 is in the tenths place?

To identify decimal place value, we use the decimal point as our reference point instead of the last number to the right. The tenths place is the first place to the right of the decimal point. The digit in the tenths place is **8**.

••••••

Which digit is in the hundredths place in each of these two decimal numbers?

a. 4.37

b. 4.370

We focus on the decimal point. The hundredths place is the second place to the right of the decimal point.

- a. The second place to the right of the decimal point in 4.37 is 7.
- **b.** The second place to the right of the decimal point in 4.370 is also **7.**

Notice in Example 2 that each digit in 4.37 holds the same place in 4.370.



Visit www. SaxonMath.com/ Int4Activities for a calculator activity. Example 2

Example 1

Thinking Skill	dths		
Verify How many hundredths	ones tenths hundred thousan		
are equal to one tenth?	4.3 7		
How many	4.370		
are equal to one hundredth?	Because the zero in the thousandths place of 4.370 does not add value. 4.37 and 4.370 are equal.		
	4.37 = 4.370		
Exampl	le 3		
	Compare: 23.25 () 23.250		
	We will write the numbers with the decimal points lined up and will compare the numbers place by place.		
	23.25		
	23.250		
	Both numbers have the same digits in the same places. The zero in the thousandths place of 23.250 adds no value, so the numbers are equal.		
	23.25 = 23.250		
When performing decimal arithmetic, it is often helpful to attach one or more zeros to the end of a decimal number, as we see below. The attached zeros do not add value, so the original problem remains the same.			
Exampl	le 4		
	Subtract: 4.37 – 1.146		
	We line up the decimal points whenever 4.37 we add or subtract decimal numbers. This -1.146 ensures that we add or subtract digits with -1.146		
	the same place values. In this example, notice that there is no digit to subtract the 4.370		
	6 from. We may fill the empty place with a -1.146		
	zero because 4.370 equals 4.37. Then we 3.224 can subtract. The answer is 3.224 .		
Lesson Practi	ce) a. Which digit in 4.370 is in the hundredths place?		
	b. Which digit in 4.370 is in the same place as the 2 in 15.24?		
	c. Name the place value of the 4 in the number 1.234.		

 d. Multiple Choice
 Which two numbers below are equal?

 A 12.34
 B 12.340
 C 1.234
 D 123.4

 e. Compare: 3.25
 32.50

 f. Compare: 3.250
 3.25

 Subtract. Show your work for each problem.

 g. 12.34 – 1.234
 h. 1.2 – 0.12

 Distributed and Integrated

*1. **Analyze** Three quarters, four dimes, two nickels, and seven pennies is how much money?

- *2. Analyze Colvin separated the 37 math books as equally as possible into 4 stacks.
 - a. How many stacks had exactly 9 books?
 - b. How many stacks had 10 books?
- ***3.** (⁽⁸³⁾ Gregory paid \$1 for a folder and received 52¢ in change. If the tax was 3¢, how much did the folder cost without tax? Explain your thinking.
- **4.** Ryan wrote each of his 12 spelling words five times. In all, how many words did he write?
- **5. Estimate** In the 2004 presidential election, 5992 voters in Blaine County, Idaho, voted for candidate John Kerry, and 4034 voters voted for candidate George Bush. Estimate the total number of votes those two candidates received and explain your estimate.

6. What is the tally for 10? (*inv.* 7)

- 7. Name the shaded part of this square
 - a. as a fraction.
 - **b.** as a decimal number.



How many juice glasses can Emma fill? Explain your answer.

*27. Multiple Choice Use the polygons below to answer parts a-c. (63, 79) A B C D D

- a. Which of these polygons has no lines of symmetry?
- b. Which two of these polygons have rotational symmetry?
- c. Which polygon is not a quadrilateral?
- ***28.** How many degrees does the minute hand of a clock turn in half an hour?
- ***29.** Compare: 4.2 () 4.200

***30.** Use the pictograph below to answer parts **a-c.**

-	•
(Inv.	6)

Animal	Typical Weight (in pounds)	
Alligator		
Porpoise		
Wild Boar		
Seal		
Key:		

- a. What amount of weight does each symbol represent?
- **b.** Write the typical weights of the animals in order from least to greatest.
- **c. Connect** Write a sentence that compares the weights of two animals.



Connection

Gas is sold in amounts to the tenth of a penny, which is the same as the thousandth of a dollar. Gus's Grand Gas Station is selling gas for \$2.679 per gallon.

- **a.** Which digit is in the thousandths place? Which digit is in the tenths place?
- **b.** Which is more: 2.679 or 2.67?
- **c.** If you were to buy ten gallons of gas, how much would you spend? If you were to pay \$30 for the gas, what would your change be?



Classifying Quadrilaterals

Power Up	
facts	Power Up I
mental	Find half of a product in a-c.
math	a. Number Sense: half of 10×12
	b. Number Sense: half of 10×24
	c. Number Sense: half of 10×480
	 d. Money: The art supplies cost \$17.50. Adam paid with a \$20 bill. How much change should he receive?
	e. Estimation: About what percent of the circle is shaded? About what percent of the circle is not shaded?
	f. Calculation: 25% of 40, \times 2, + 4, \div 3
	g. Roman Numerals: Write XI in our number system.
problem solving	Choose an appropriate problem-solving strategy to solve this problem. Below we show the first five terms of a sequence. The terms of the sequence increase from left to right. Estimate how many terms will be in the sequence when it reaches a number that is 500 or greater. Then check your estimate by continuing the sequence until you reach a number that is 500 or greater.
	1. 2. 4. 8. 16

New Concept

Recall from Lesson 63 that a quadrilateral is a polygon with four sides. In this lesson we will practice recognizing and naming different types of quadrilaterals. On the following page, we show four different types.



Verify

A parallelogram is a quadrilateral with two pairs of parallel sides. Figures A, B, C, and D each have two pairs of parallel sides, so all four figures are parallelograms. A trapezoid is a quadrilateral with exactly **one** pair of parallel sides. Figure *E* is not a parallelogram; it is a trapezoid.

square

A rectangle is a special type of parallelogram that has four right angles. Figures C and D are rectangles. A **rhombus** is a special type of parallelogram whose sides are equal in length. Figure B is a rhombus, as is figure *D*. A **square** is a regular quadrilateral. Its sides are equal in length, and its angles are all right angles. Figure D is a square. It is also a parallelogram, a rhombus, and a rectangle.



Example 3



Model

Find a quadrilateral in your classroom. Identify and describe the parallel, perpendicular, and intersecting segments in the quadrilateral.







We will consider the relationships between the sides of each quadrilateral.

- A The opposite sides are parallel, and the adjacent sides are perpendicular.
- **B** The opposite sides are parallel, and the adjacent sides intersect but are not perpendicular.
- **C** There are no parallel or perpendicular sides.
- **D** One pair of opposite sides is parallel, and another side is perpendicular to the parallel sides.

Only **figure C** has sides that are not parallel or perpendicular.

Example 4

Describe the angles in each of the quadrilaterals in Example 3.

Figure A is a square; it has four right angles.

Figure **B** is a parallelogram; it has **two acute angles and two obtuse angles.**

Figure C is a quadrilateral; it has **two obtuse angles and two acute angles.**

Figure **D** is a trapezoid; it has **two right angles**, **one acute angle**, **and one obtuse angle**.



Quadrilaterals in the Classroom

Look around the room for quadrilaterals. Find examples of at least three different types of quadrilaterals illustrated in the beginning of this lesson. Draw each example you find, and next to each picture, name the object you drew and its shape. Then describe how you know that the object is the shape you named and describe the relationships of the sides of each quadrilateral.

Activity 2

Symmetry and Quadrilaterals

Materials needed:

- Lesson Activity 41
- mirror or reflective surface

If a figure can be divided into mirror images by a line of symmetry, then the figure has reflective symmetry. A mirror can help us decide if a figure has reflective symmetry. If we place a mirror upright along a line of symmetry, the half of the figure behind the mirror appears in the reflection of the other half. Use a mirror to discover which figures in **Lesson Activity 41** have reflective symmetry. If you find a figure with reflective symmetry, draw its line (or lines) of symmetry.

Lesson Practice

Classify Describe each quadrilateral as a trapezoid, parallelogram, rhombus, rectangle, or square. (More than one description may apply to each figure.)



 e. Describe the angles in figures a–d and the relationships between the sides. f. Draw two parallel line segments that are the same length. Then make a quadrilateral by drawing two more parallel line segments that connect the endpoints. Is your quadrilateral a parallelogram? Why or why not?

Distributed and Integrated

*1. Analyze Use this information to answer parts **a**–**c**. *Lanisha invited 14 friends over for lunch. She plans to make 12 tuna sandwiches, 10 bologna sandwiches, and 8 chicken sandwiches.*

- a. How many sandwiches will Lanisha make in all?
- b. Including Lanisha, each person can have how many sandwiches?
- **c.** If Lanisha cuts each tuna sandwich in half, how many halves will there be?
- **2.** Five pounds of grapes cost \$2.95. What is the cost per pound?

***3.** If each side of a hexagon is 4 inches long, what is the perimeter of the ^{(Inv. 2,} ₆₃₎ hexagon in feet?

- **4. Represent** Nine million, four hundred thousand is how much greater than two million, seven hundred thousand?
 - ***5.** Three brands of whole grain cereal cost \$4.68, \$4.49, and \$4.71. ⁽³⁰⁾ Arrange these prices in order from least to greatest.
 - *6. Estimate Lauren saw that a package of 50 blank CDs costs \$9.79. (58) Estimate the cost for Lauren to buy 100 blank CDs. Explain your answer.
- **7.** Name the shaded part of the large square (ny, 4)
 - a. as a fraction.

Written Practice

b. as a decimal number.



8. **Represent** Use words to write $7572\frac{1}{8}$.

- *9. **Represent** At Kelvin's school, one fifth of the 80 fourth grade students ride the bus to and from school each day. How many fourth grade students ride the bus? Draw a picture to illustrate the problem.
- 10. How many different three-digit numbers can you write using the digits
 ⁽³⁾ 9, 1, and 5? Each digit may be used only once in every number you write.
- **11.** Franca's trip only lasted for a couple of hours. According to the clocks shown below, exactly how long did the trip take?



- * **12.** Justify James traveled 301 miles in 7 hours. He traveled an average of how many miles per hour? Explain why your answer is reasonable.
- * **13.** Martino bought 3 folders priced at \$1.99 each. Sales tax was 33¢. He paid with a \$20 bill. How much money should he get back?

14.
$$\$25 + \$2.75 + \$15.44 + 27¢$$
*** 15.** $m + 0.26 = 6.2$ **16.** $\$100 - \89.85 **17.** 60×900 **18.** 42×30 **19.** 21×17 *** 20.** 36 $\times 74$ (90) $\times 74$ *** 21.** 48 (90) $\times 25$ *** 23.** $9)\overline{918}$ **24.** $5r = 485$ **25.** $6)\overline{482}$ **26.** $\$50.00 \div 8$

27. 2100 ÷ 7

*29. Classify This polygon is what type of quadrilateral? How do you know?



***30. Represent** Draw and shade rectangles to show that $1\frac{2}{3}$ equals $\frac{5}{3}$.



Stephanie's class has to identify polygons for math class.

- **a.** Draw and label a picture for each of the following shapes that her class could use as examples: square, rectangle, rhombus, trapezoid, and parallelogram.
- **b.** Explain why each drawing fits its name.



Power Up

Estimating Multiplication and Division Answers

facts	Power Up I		
mental	Find half of a product in a-c.		
math	a. Number Sense: half of 10×18		
	b. Number Sense: half of 10×44		
	c. Number Sense: half of 10×260		
	d. Time: How many minutes are in $1\frac{1}{2}$ hours?		
	e. Measurement: How many quarts is 3 gallons?		
	f. Estimation: About how many feet is 1989 yards?		
	g. Calculation: 3^2 , + 1, × 5, - 1, $\sqrt{-}$		
	h. Roman Numerals: Write 9 in Roman numerals.		
problem solving	Choose an appropriate problem- solving strategy to solve this problem. In this Venn diagram, the circle on the left stands for animals with the ability to fly, and the circle on the right stands for birds. The <i>R</i> in the overlapping portion of the circles stands for robins, which are birds that can fly. The <i>O</i> stands for ostriches, which are birds that cannot fly. The <i>B</i> stands for bats, which can fly but are not birds. The <i>W</i> stands for whales, which are not birds and cannot fly. Copy the Venn diagram on your paper, and place an abbreviation for a penguin, eagle, goldfish, and cat.		



Estimation can help prevent mistakes. If we estimate the answer before we multiply, we can tell whether our answer is reasonable.

Example 1

Luke multiplied 43 by 29 and got 203. Is Luke's answer reasonable?

We estimate the product of 43 and 29 by multiplying the rounded numbers 40 and 30.

 $40 \times 30 = 1200$

Luke's answer of 203 and our estimate of 1200 are very different, so Luke's answer is **not reasonable.** He should check his work.

Discuss What is the exact product? Is the exact product close to 1200?

Example 2

Estimate the product of 38 and 53. Then find the exact answer.

We estimate the product by multiplying the rounded numbers 40 and 50.

$$40 \times 50 = 2000$$

Then we find the exact answer.

38	
× 53	
114	
190	
2014	

Our estimate of the product was **2000**, so our answer of **2014** is reasonable.

Example 3

Thinking Skill

Estimate the quotient of 1845 divided by 6.

Estimate How would you estimate the quotient of 184 ÷ 6? We choose a number close to 1845 that can easily be divided by 6. We know that 18 is a multiple of 6, so 1800 is a compatible dividend. We can calculate mentally: "18 hundred divided by 6 is 3 hundred."

1800 ÷ 6 = **300**



- *9. **Represent** One half of the 32 chess pieces were still on the board. ⁽⁷⁰⁾ How many chess pieces were still on the board? Draw a picture to illustrate the problem.
- **10.** Miriam left home at 10:30 a.m. She traveled for 7 hours. What time was it when she arrived?
- **11.** Maureo traveled 42 miles in 1 hour. If he kept going at the same speed, how far would he travel in 20 hours?
- * **12.** Violet gave the cashier \$40 for a toaster that cost \$29.99 plus \$1.80 in tax. What was her change? Write one equation to solve the problem.
- * **13.** Alvin faced the sun as it set in the west, then turned 90° counterclockwise and headed home. In what direction was Alvin heading after the turn?

14. $n + 8 + 2 + 3 + 5 + 2 = 24$	15. $4.12 - (3.6 + 0.2 + 0.125)$
16. \$18 − \$15.63	17. \$15.27 + \$85.75
18. $2^3 \times \sqrt{25}$	19. 30 × 90
20. \$7.50 × 8	*21. 49 (⁹⁰⁾ × 62
* 22. 54 (90) × 23	23. 74 (67) × 40
24. 4)\$6.36	25. 5)800
26. 473 ÷ 8	27. 3 <i>m</i> = 1800

***28.** Estimate the quotient when 1520 is divided by 5. Then find the exact quotient.

*29. **Represent** Draw and shade circles to show that $2\frac{1}{4}$ equals $\frac{9}{4}$.

30. Find the perimeter and area of this rectangle. (Inv. 2, Inv. 3)

50 ft 20 ft



Two-Step Word Problems

Power Up

facts	Power Up H
mental math	Five is half of 10. To multiply by 5, we can multiply by half of 10. For example, 5×12 equals half of 10×12 . Find each product by multiplying by "half of 10" in a–d.
	a. Number Sense: 5×16
	b. Number Sense: 5×24
	c. Number Sense: 5×28
	d. Number Sense: 5×64
	e. Measurement: A <i>stone</i> is a British unit of weight equal to 14 pounds. Two stone is 28 pounds, 3 stone is 42 pounds, and so on. How many pounds is 10 stone?
	f. Estimation: Lydia walked 1 km in 608 seconds. About how many minutes did it take her to walk 1 km?
	g. Calculation: 10% of 40, \times 10, $+$ 5, \div 5
	h. Roman Numerals: Write XIV in our number system.
problem solving	Choose an appropriate problem-solving strategy to solve this problem. Find the next five numbers in this sequence. Then describe the sequence in words.
	, 64, 32, 16, 8,,,,,
New Concept	

We have practiced two-step word problems that involve finding total costs (including tax) and change back. Starting with this lesson, we will practice other kinds of two-step problems. Writing down the given information and using problem-solving strategies is often helpful in solving these problems.

Example 1



Jim is 5 years older than Ali. Ali is 2 years younger than Blanca. Blanca is 9 years old. How old is Jim?

We will use two steps to solve the problem. First we will use Blanca's age to find Ali's age. Then we will use Ali's age to calculate Jim's age. We write down the given information.

Blanca is 9 years old.

Ali is 2 years younger than Blanca.

Jim is 5 years older than Ali.

We know that Blanca is 9 years old. Ali is 2 years younger than Blanca, so Ali is 9 - 2, or 7 years old. Jim is 5 years older than Ali, so Jim is 7 + 5, or **12 years old.**

Example 2

Thinking Skill

What are the two

steps needed to find the cost of

each pound?

Verify

Ja'Von paid for 5 pounds of apples with a \$10 bill. His change was \$6. What was the cost of each pound of apples?

We begin by finding how much all 5 pounds of apples cost. If Ja'Von paid for the apples with a \$10 bill and received \$6 in change, then all 5 pounds must have cost \$4.

- \$10 amount paid
- \$ 6 change
 - \$ 4 cost of 5 pounds of apples

To find the cost of each pound of apples, we divide \$4 by 5.



Each pound of apples cost **\$0.80**.

Example 3

One of the foods that Maribella feeds her pet rabbit is 2 ounces of lettuce each day. In how many days does her rabbit eat a pound of lettuce? How many pounds of lettuce does the rabbit eat in 4 months?

A pound is 16 ounces. At 2 ounces per day, the rabbit eats a pound of food every **8 days.**

$$16 \div 2 = 8$$

A month is about 30 days, so 4 months is 4×30 days, which is 120 days. We divide 120 days into groups of 8 days to find the number of pounds of lettuce the rabbit eats.

$$120 \div 8 = 15$$

In 4 months, the rabbit eats about **15 pounds** of lettuce.

Example 4

Point B represents which number on this number line?



Sometimes two-step problems are not word problems. We can solve problems like this with two or three steps of arithmetic.

We see that the distance from point A to point C is 100.

Step 1: 200 - 100 = 100

The distance is divided into 4 segments. By dividing 100 by 4, we find that each segment is 25.

Step 2: 100 ÷ 4 = 25

Step 3: If we count by 25s from 100, point *A* to point *B*, we find that point *B* represents **175.** Since point *B* is one segment from point *C*, we can check the answer by counting back 25 from 200. The result is 175, which is our original answer.

Discuss How could we use the *guess and check* strategy to solve this problem?

Example 5

If y = 2x + 1, then what is y when x = 3?

The equation y = 2x + 1 shows us how to find the number that y equals when we know what the number x equals.

The equation means, "To find *y*, multiply *x* by 2 and then add 1." In this equation x is 3, so we multiply 2 times 3 and then add 1.

$$y = (2 \times 3) + 1$$

 $y = 6 + 1$
 $y = 7$

When *x* is 3, *y* is **7.**

Represent What is *y* when *x* is 5?

We can write these values in a table to find the answer.



1. Joel gave the clerk a \$5 bill to pay for a half gallon of milk that cost
 \$1.06 and a box of cereal that cost \$2.39. How much change should he receive?

*2. Eighty-one animals live at the zoo. One third of them are not mammals. (70, 94) The rest are mammals. How many mammals live at the zoo? (*Hint:* First find the number of animals that are not mammals.) **3.** Ciante planted 8 rows of apple trees. There were 15 trees in each row. How many trees did she plant?

4. A ruble is a Russian coin. If four pounds of bananas costs one hundred and fifty-six rubles, what is the cost in rubles of each pound of bananas?

- *5. a. This scale shows a mass of how many grams?
 - **b. Explain** Would this fruit have the same mass on another planet? Explain why.



*6. Felix is ten years younger than Zatravian. Zatravian wrote this formula for finding Felix's age: F = Z - 10. Find F when Z is 27.

- **7.** Name the shaded part of the large square *(Inv. 4)* **a.** as a fraction.
 - **b.** as a decimal number.



0 Grams

200

800

***8.** Estimate the product of 32 and 48. Then find the exact product. (93)

9. Represent Bactrian camels have 2 humps. One third of the 24 camels were Bactrian. How many camels were Bactrian? Draw a picture to illustrate the problem.

10. A quart is a quarter of a gallon. A quart is what percent of a gallon? $I_{\text{INV}, 5}^{(40)}$

* **11. Classify** For each statement, write either "true" or "false."

- **a.** Every square is also a rectangle.
- **b.** Every rectangle is also a square.

- *12. a. **Represent** Four hundred seventy-one of the one thousand students in the school were girls. Girls made up what fraction of the students in the school?
 - **b. Represent** Write your answer for part **a** as a decimal number. Then use words to name the number.
- *** 13.** Which digit in 1.875 is in the tenths place?
- * **14.** If y = 2x-3, what is y when x is 5?
 - **15.** Tyler traveled 496 miles in 8 hours. He traveled an average of how many miles per hour?
- *** 16.** Find 8.3 (1.74 + 0.9). Describe the steps in order. (45, 91)

17. 63 × 1000	18. 80 × 50¢	19.	37
(65)	(80)	(17)	81
* 20. 52	*21. 36		45
⁽⁹⁰⁾ × 15	⁽⁹⁰⁾ × 27		139
	<u> </u>		7
			15
22. 2)714	23. 6) 789	+	60

24. 3n = 624 **25.** $5 + w = 5^2$

- ***26. Represent** Draw and shade rectangles to show that $1\frac{2}{5}$ equals $\frac{7}{5}$.
- **27.** A room is 5 yards long and 4 yards wide. How many square yards of carpeting are needed to cover the floor?
- **28.** The radius of this circle is 15 millimeters. The diameter of the circle is how many centimeters?



***29.** a. **Verify** Which of these letters has two lines of symmetry? (79, 81)

VWXYZ

- b. **Verify** Which two letters have rotational symmetry?
- **c. Multiple Choice** The angle formed by the letter V measures about how many degrees?

A 45°	B 90°	C 135°	D 180°

* **30.** Rihanne and Kendra sat across the table from each other in the café. ⁽⁷³⁾ When Rihanne was finished looking at the one-page menu, she moved it over to Kendra's side of the table so Kendra could read it. Which two transformations did Rihanne use to move the menu?





A long mountain trail has an upward elevation of 4780 feet.

- **a.** If you were to hike the trail five hours each day for four days, how many feet would be the average elevation gain each day?
- **b.** What would be the average elevation gain for each hour of hiking?



Power Up

Two-Step Problems About a Fraction of a Group

facts Power Up H mental Find each product by multiplying by "half of 10" in **a-c.** math **a.** Number Sense: 5×46 **b.** Number Sense: 5×62 c. Number Sense: 5×240 **d.** Money: The price of the blouse is \$24.87. Sales tax is \$1.95. What is the total cost? e. Measurement: The large glass of water weighed half a kilogram. How many grams is half a kilogram? f. Estimation: The package of 10 pencils costs \$1.98. Round that price to the nearest dollar and then divide by 10 to estimate the cost per pencil. g. Calculation: $\sqrt{4}$, \times 7, + 1, + 10, $\sqrt{-4}$, - 4 h. Roman Numerals: Write 15 in Roman numerals. problem Choose an appropriate problem-solving strategy to solve this solving problem. On February 4, Edgar remembered that his two library books were due on January 28. The fine for late books is 15¢ per book per day. If he returns the books on February 4, what will be the total fine? New Concept

The word problems in this lesson are two-step problems involving fractions of a group. First we divide to find the number in one part. Then we multiply to find the number in more than one part.

Example 1

Thinking Skill

Verify

What are the two steps needed to find the number of campers who wore green jackets?

There were 30 campers in the state park. Two thirds of them wore green jackets. How many campers wore green jackets?

The word *thirds* tells us there were 3 equal groups. First we find the number of campers in each group. Since there were 30 campers altogether, we divide 30 by 3.

10 3)30

There were 10 campers in each group. We draw this diagram:



Two thirds wore green jackets. In two groups there were 2×10 campers, or **20 campers** who wore green jackets. We also see that one group did not wear green jackets, so 10 campers did not wear green jackets.

Example 2

The force of gravity on Mars is about $\frac{2}{5}$ the force of gravity on Earth. A rock brought back to Earth from Mars weighs 50 pounds. How much did the rock weigh on Mars?

The mass of the rock is the same on Earth as it was on Mars because it is the same amount of rock. However, Earth is more massive than Mars, so the force of gravity is greater on Earth. The rock on Mars weighed only $\frac{2}{5}$ of its weight on Earth. To find $\frac{2}{5}$ of 50 pounds, we first find $\frac{1}{5}$ of 50 pounds by dividing 50 pounds by 5.

50 pounds \div 5 = 10 pounds



Each fifth is 10 pounds, so $\frac{2}{5}$ is 20 pounds. We find that the rock that weighs 50 pounds on Earth weighed only **20 pounds** on Mars.

Discuss Why did the weight of the rock change when it was brought to Earth? How does the mass of the rock on Mars compare to the mass of the rock on Earth?

Represent Diagram each problem. Then answer the question.

- **a.** Three fourths of the 24 checkers were still on the board. How many checkers were still on the board?
- **b.** Two fifths of 30 students studied more than one hour for a test. How many students studied for more than one hour?
- **c.** The force of gravity on Mercury is about $\frac{1}{3}$ the force of gravity on Earth. How much less would a tire weigh on Mercury if it weighs 42 lb on Earth? Would the mass be the same? Why or why not?
- **d.** Explain the difference between weight and mass.

Written Practice

Distributed and Integrated

*1. **Interpret** Use this tally sheet to answer parts **a-c**. (Inv. 7)

Results of Class Election		
Candidate	Tally	
Irma	JHT	
Hamish	JHT I	
Thanh	JHT III	
Marisol	JHT JHT	

- a. Who was second in the election?
- **b.** Who received twice as many votes as Hamish?
- **c.** Altogether, how many votes were cast?

2. Write these amounts in order from greatest to least: (Inv. 4)

\$1.45 \$2.03 \$0.99 \$1.48

3. Formulate The Osage River in Kansas is 500 miles long. The Kentucky River is 259 miles long. How many miles longer is the Osage River? Write and solve an equation.

*4. (Represent) Two fifths of the 20 balloons were yellow. How many balloons were yellow? Draw a picture to illustrate the problem.

- ***5.** Tim is 5 years younger than DeMario. DeMario is 2 years older than Lucinda. Lucinda is 11 years old. How old is Tim? How did you find your answer?
- 6. Name the shaded part of this group (Inv. 4, 74)
 - a. as a fraction.
 - **b.** as a decimal number.
- **7.** The fraction $\frac{1}{10}$ equals 10%. What percent of the group in problem **6** is shaded?
 - *8. Estimate the product of 88 and 59. Then find the exact product.
 - 9. Sue's birthday is May 2. Her birthday will be on what day of ⁽⁵⁴⁾ the week in the year 2045?

MAY 2045							
	S	М	Т	W	Т	F	S
		1	2	3	4	5	6
	7	8	9	10	11	12	13
	14	15	16	17	18	19	20
	21	22	23	24	25	26	27
	28	29	30	31			

*10. Point W represents what number on this number line? 700 600 **12.** \$86.45 - (\$74.50 + \$5) **11.** \$32.63 + \$42 + \$7.56 **15.** $9^2 - \sqrt{81}$ **13.** 83 × 40 **14.** 1000 × 53 (67) (85) * **17.** br (90) × 32 ***16.** 5-(90) × 16 18. \$8.95 (58) \times 4 **20.** 4)714 **21.** 6) 1385 **19.** 3)625 **22.** $\frac{900}{5}$ **24.** 8*m* = \$28.56 **23.** 3748 ÷ 9



- *25. **Represent** This circle shows that $\frac{2}{2}$ equals 1. Draw a circle that shows that $\frac{3}{3}$ equals 1.
- **26.** Find the perimeter and area of this rectangle. (Inv. 2, Inv. 3)



- ***27. a.** Draw a quadrilateral that is congruent to the quadrilateral (66, 79) at right. Then write the name for this type of guadrilateral.
 - **b.** Draw the line of symmetry on the figure you created.
- ***28.** Compare: 0.05 (0.050) **Explain** Kelly ran and jumped 9 ft 6 in. How many inches did *29. (Inv. 2, 94) Kelly jump?
- ***30.** The table shows the relationship between the number of hours Aidan (Inv. 8) works and the amount of money he earns.

Number of Hours Worked	Income Earned (in dollars)
1	19
2	38
3	57
4	76
5	95

- Generalize Write a word sentence that describes the a. relationship of the data.
- Predict Aidan works 40 hours each week. What is a reasonable b. estimate of the amount of income he earns each week? Explain your answer.





Average

Power Up

SSON

facts Power Up H mental Find half of a product in **a–c**. math **a. Number Sense:** half of 100×12 **b.** Number Sense: half of 100×24 **c.** Number Sense: half of 100×48 **d.** Money: The salad cost \$4.89. Ramona paid for it with a \$10 bill. How much change should she receive? e. Geometry: The angles of the triangle measured 47°, 43°, and 90°. What is the sum of the angle measures? f. Estimation: In 10 minutes, Tevin counted 25 cars that drove through the intersection. About how many cars might Tevin expect to count in 20 minutes? g. Calculation: $16 \div 2, -6, \times 2, \sqrt{}$ **h.** Roman Numerals: Write XXXIII in our number system. problem Choose an appropriate problem-solving strategy to solve this solving problem. There are three light switches that each control a row of lights in the classroom—a row in front, a row in the middle, and a row in back. Make a tree diagram to find the different ways the rows of lights can be turned on or off. Use the tree diagram to count the total number of combinations. **New Concept**



Here we show three stacks of coins:



There are 15 coins in all. If we rearrange the coins to have an equal number in each stack, we get 5 coins in each stack.



We say that the *average* number of coins in each stack is 5. Finding an average is a two-step process. First we find how many there are altogether. Then we find how many there would be in each group if the groups were equal.



- **b. Analyze** There were two stacks of books on the shelf: one with 17 books and the other with 11 books. Allison moved some books from the taller stack to the shorter stack so that the numbers of books in the two stacks were equal. When she finished, how many books were in each stack?
- **c.** Spencer's scores on his first three games were 85, 85, and 100. What was the average of his first three game scores?

Distributed and Integrated

- *1. Analyze Freddie is 2 years older than Francesca. Francesca is twice as old as Chloe. Chloe is 6 years old. How old is Freddie?
- *2. (Analyze) What is the total number of days in the first three months of a leap year?
- *3. It costs \$1.52 to mail the package. Nate put three 37¢ stamps on the package. How much more postage does Nate need to mail the package?
- *4. Thirty-two desks were arranged as equally as possible in 6 rows.
 a. How many rows had exactly 5 desks?
 - b. How many rows had 6 desks?

Written Practice

- *5. **Represent** Two thirds of the 21 riders rode their horses bareback. How many riders rode bareback? Draw a picture to illustrate the problem.
- ***6. a.** What decimal number names the shaded part of the large square at right?
 - b. What decimal number names the part that is not shaded?
 - c. What percent of the square is shaded?



***7. Explain** Near closing time, 31 children and adults are waiting in line to board a ride at an amusement park. Eight people board the ride at one time. How many people will be on the last ride of the day? Explain your answer.

8. Round 3874 to the nearest thousand. (54)

9. Estimate Alicia opened a liter of milk and poured half of it into a

^{(40,} Inv. 5) pitcher. About how many milliliters of milk did she pour into the pitcher? What percent of the milk was still in the container?

10. The sun was up when Mark started working. It was dark when he stopped working later in the day. How much time had gone by?



- * **11.** For five days Pilar recorded the high temperature. The temperatures were 79°F, 82°F, 84°F, 81°F, and 74°F. What was the average high temperature for those five days?
 - **12.** (60) Leena drove 368 miles in 8 hours. If she drove the same number of miles each hour, how far did she drive each hour? Explain how you found your answer.

13.	496,325	14.	\$36.00		15.	\$12.45
(51)	+ 3,680	(52)	- \$30.78		(22)	\$ 1.30
						\$ 2.00
+ 40	00	+ 47	05			\$ 0.25
* 16. (90)	26	* 17. (90)	25			\$ 0.04
	<u>× 24</u>		$\times 25$			\$ 0.32
					+	\$ 1.29
18. (41, 80)	8 <i>m</i> = \$16.40	19. (86)	60 × 300			
20. (58)	\$8.56 × 7	21. 7)845	2	22. 9)1000		23. $\frac{432}{6}$
*24.	Represent Drav	w and shade	a circle that	shows that $\frac{4}{4}$ e	equals	1.

(89)

25. The wall was 8 feet high and 12 feet wide. How many square feet of wallpaper were needed to cover the wall?

26. (94, 96) Below are Tene's scores on the first seven games. Refer to the scores below to answer parts a-c.

85, 85, 100, 90, 80, 100, 85

- **a.** Rearrange the scores so that the scores are in order from lowest to highest.
- **b.** In your answer to part **a**, which score is the middle score in the list?
- **c.** In the list of game scores, which score occurs most frequently?

*27. **Estimate** What is a reasonable estimate of the number in each group when 912 objects are separated into 3 equal groups? Explain why your estimate is reasonable.

- *28. According to many health experts, a person should drink 64 ounces of water each day. If Shankeedra's glass holds 8 ounces of water, how many glasses of water should she drink in one day?
- *29. Arthur told his classmates that his age in years is a single-digit odd number greater than one. He also told his classmates that his age is not a prime number. How old is Arthur?

*30. If y = 3x - 1, what is y when x is 2?



Mylah decided she wanted to grow three peanut plants for her science class. After five months, one of her plants is 1 ft 6 in. tall. The second plant is 1 ft 2 in. tall, and the third is 10 inches tall.

- **a.** Convert the heights of the first two peanut plants to inches, and then find the average height of all of Mylah's plants (in inches).
- **b.** Convert the average to feet and inches.



Power Ud

Mean, Median, Range, and Mode

facts Power Up H mental Fifty is half of 100. Find each product by multiplying by half math of 100 in a-d. a. Number Sense: 50×16 **b.** Number Sense: 50×44 c. Number Sense: 50×26 d. Number Sense: 50×68 e. Money: The groceries cost \$32.48 and the magazine cost \$4.99. What was the total cost? f. Estimation: Each box is 30.5 cm tall. Estimate the height (using cm) of a stack of 6 boxes. **g. Calculation:** $200 \div 2, \div 2, \div 2$ h. Roman Numerals: Write 25 in Roman numerals. problem Choose an appropriate problem-solving strategy to solve this solving problem. There are 365 days in a common year, which is about 52 weeks. However, since 52 weeks is exactly 364 days, a year does not start on the same day of the week as the start of the preceding year. If a common year starts on a Tuesday, on what day of the week will the following year begin?

New Concept

In Lesson 96 we practiced finding an average. To find the average of a set of numbers, we added the numbers and then divided by the number of numbers. Another name for the average is the **mean**.

Example 1

Find the mean of lan's seven game scores.

80, 85, 85, 10, 90, 90, 85

We add the scores and divide by the number of scores. The sum of the scores is 525. We divide the sum by 7.

 $525 \div 7 = 75$

The mean, or average, of the seven scores is **75.** This means that lan's seven game scores are equivalent to seven scores of 75. This might seem low since six of lan's scores were higher than 75. However, his one very low score of 10 lowered his average.

The **median** of a set of numbers is the middle number when the numbers are arranged in order of size. When there is an even set of numbers, the *median* is the average of the two middle numbers.

```
Example 2
```

Find the median of lan's seven game scores.

80, 85, 85, 10, 90, 90, 85

The median score is the middle score. To find the median score, we arrange the scores in order. We begin with the lowest score.

10, 80, 85, 85, 85, 90, 90 3 scores 3 scores middle

We see that the median score is 85.

Discuss Explain how to find the median of the following set of numbers.

5, 4, 3, 8, 7, 7

Notice that the low score of 10 does not affect the median. A score that is far from the other scores is called an **outlier**. *Outliers* sometimes affect the mean while having little or no effect on the median. Below we have placed these scores on a line plot. We see that most of the scores are close together.



The outlier is far away from the other scores.





d. Every X on this line plot stands for the age of a child attending a party. How many children attended the party? What is the mode of the ages?



- **b.** How many children were *not* swimming?
- c. How many children were either hiking or skating?
- d. How many more children were flying kites than were swimming?
- *2. **Represent** Three fourths of the one thousand gold coins were special gold coins called doubloons. How many doubloons were there? Draw a picture to illustrate the problem.

***3.** What percent of the gold coins in problem **2** was doubloons? $\frac{(Inv. 5, g_5)}{g_5}$

4. Write each mixed number as a decimal: $\binom{(d)}{(d)}$

a.
$$3\frac{5}{10}$$
 b. $14\frac{21}{1000}$ **c.** $9\frac{4}{100}$

*5. Estimate the product of 39 and 406. Then find the exact (93) product.

***6.** If y = 4x - 2, what is y when x is 4?

***7.** Write these fractions in order from least to greatest: (Inv. 9)

> <u>1</u> 2 <u>5</u> 8 $\frac{3}{4}$

8. Compare: 2 thousand () 24 hundred (33

Refer to the rectangle at right to answer problems 9 and 10.

- 9. What is the perimeter of the rectangle (Inv. 2,
 - 69) a. in millimeters?
 - **b.** in centimeters?
- 10. What is the area of the rectangle (Inv. 3, 86)
 - a. in square millimeters?
 - **b.** in square centimeters?
- 11. Santos figured the trip would take seven and a half hours. He left (27) at 7 a.m. At what time does he think he will arrive?
- *12. Analyze What is the average (mean) number of days per month in the (96) first three months of a common year?

13. 25 × 40	14. 98¢ $ imes$ 7
15. $\sqrt{36} \times \sqrt{4}$	16. $\frac{3^3}{3}$



17.	36	* 18.	35	19.	4
(90)	imes 34	(90)	\times 35	(2)	2
					1
00	0.000	<u> </u>	01 6 1024		3
20. (41, 76)	8/// =	\$70.00	(80)		4
					7
22.	800 ÷	7	23. 487 ÷ 3		2
(00)			(03)		2
24.	\$2.74	+ \$0.27 + \$6 + 49¢			3
(43)	•	T - T T			4
25	0 /07	(27 + 226)		+	- <u>X</u>
25. (45, 50)	5.407	$-(0.7 \pm 2.00)$		2	42

*26. **Represent** Draw and shade circles to show that $2\frac{1}{3}$ equals $\frac{7}{3}$.

*27. Analyze Listed below are the number of points Amon scored in his last nine basketball games, which range from 6 to 10. Refer to these scores to answer parts a-e.

8, 7, 7, 8, 6, 10, 9, 10, 7

- a. What is the mode of the scores?
- **b.** What is the median of the scores?
- c. What is the range of the scores?
- d. What is the mean of the scores?
- e. Are there any outliers?
- **28.** Each school day, Brent's second class begins at 9:00 a.m. What ⁽⁸⁷⁾ kind of angle is formed by the minute hand and the hour hand of a clock at that time?
- *29. **Explain** Thirty-one students are entering a classroom. The desks in the classroom are arranged in rows with 7 desks in each row. If the students fill the first row of desks, then fill the second row of desks, and so on, how many full rows of students will there be? How many students will sit in a row that is not full? Explain your answer.
- ***30.** Melvin was reading a book. When he finished reading every other page, ⁽⁷³⁾ Melvin flipped the page. Turning a page is like which transformation?

LESSON 98

• Geometric Solids

Power Up

facts

Power Up H

mental W math th

We can double one factor of a multiplication and take one half of the other factor to find a product.

Find each product by the "double and half" method in **a-d**.

- a. Number Sense: 3×14
- **b. Number Sense:** 4×16
- c. Number Sense: 5×22
- d. Number Sense: 50×24
- **e. Money:** \$1.00 42¢
- **f. Estimation:** Choose the more reasonable estimate for the height of a ceiling: 250 cm or 250 m.
- g. Calculation: 6^2 , + 4, 30, × 10
- h. Roman Numerals: Write 25 in Roman numerals.

problem solving Choose an appropriate problem-solving strategy to solve this problem. To get to the room where he will have his yearly medical checkup, Jerome will walk through three doors—a door into the doctor's office building, a door into the waiting room, and a door into the checkup room. Each door might be either open or closed when Jerome gets to it. List the possible combinations of open and closed doors that Jerome might encounter on his way into the checkup room. Use the abbreviations "O" for *open* and "C" for *closed*.



Two-dimensional figures such as triangles, rectangles, and circles are flat shapes that cover an area but do not take up space. They have length and width but not depth. Objects that take up space are things such as cars, basketballs, desks, and houses. People also take up space. Geometric shapes that take up space are called **geometric solids**. Solids have three dimensions. The chart below shows the names of some geometric solids.



Geometric Solids



Example 1



Example 2

What is the shape of a soup can?

A soup can has the shape of a cylinder.

A flat surface of a solid is called a **face**. Two faces meet at an **edge**. Three or more edges meet at a corner called a *vertex* (plural: *vertices*). The bottom face is called the **base**.



A circular cylinder has one curved surface and two flat circular surfaces. A cone has one curved surface and one flat circular surface. The pointed end of a cone is its **apex**. A sphere has no flat surfaces.

Example 3

- a. How many faces does a box have?
- b. How many vertices does a box have?
- c. How many edges does a box have?

Find a closed, rectangular box in the classroom (a tissue box, for example) to answer the questions above.

- a. 6 faces (top, bottom, left, right, front, back)
- **b. 8 vertices** (4 around the top, 4 around the bottom)
- **c. 12 edges** (4 around the top, 4 around the bottom, and 4 running from top to bottom)

Activity

Geometric Solids in the Real World

Material needed:

• Lesson Activity 42

Looking around us, we see examples of the geometric solids shown in the table of this lesson. With some objects, two or more shapes are combined. For example, in a building we might see a triangular prism and a rectangular prism.



- a. How old is Joaquin?
- **b.** How old is Jovana?
- c. When Joaquin is 16 years old, how old will Jovana be?
- *2. D'Andra bought an artichoke and 6 pounds of carrots for \$2.76. If the artichoke cost 84¢, how much did 1 pound of carrots cost? (*Hint:* First find the cost of all the carrots.)

3.	Compare. Write $>$, $<$, or $=$.	
(33)	a. 206,353 🔵 209,124	b. 518,060 🔵 518,006

4. Write these numbers in order from greatest to least:

89,611 120,044 102,757 96,720

5. **Represent** Write each mixed number as a decimal: **a.** $5\frac{31}{1000}$ **b.** $16\frac{7}{10}$ **c.** $5\frac{7}{100}$

*6. **Represent** Three fifths of the team's 40 points were scored in the first half. How many points did the team score in the first half? Draw a picture to illustrate the problem.

***7.** One fifth is 20%. What percent is three fifths? $g_{5}^{(Inv. 5, g_{5})}$

8. Represent Use words to write 7.68.

9. Represent Use words to write 76.8.

* **10.** (93) Armondo estimated that the exact product of 78 and 91 was close to 720. Did Armondo make a reasonable estimate? Explain why or why not.

11. Connect Name the number of shaded squares below

- a. as a mixed number.
- b. as a decimal.

* **12.** There were 24 people in one line and 16 people in the other line. What was the average number of people per line?

13. Makayla's school day ends 5 hours 20 minutes after the time shown on the clock. What time does Makayla's school day end?



14. Mr. Romano could bake 27 dozen whole wheat muffins in 3 hours.

- a. How many dozen whole wheat muffins could he bake in 1 hour?
- b. How many dozen whole wheat muffins could he bake in 5 hours? (*Hint:* Multiply the answer to part **a** by 5.)

15. (50)	3.65 + 4.2 + 0.625	16. \$13.70 − \$6.85		
17. (85)	26 × 100	18. 9 × 87¢	19. (90)	14 imes 16
20. (62, 90)	15 ²	21. $\frac{456}{6}$	22. (67)	47 <u>× 60</u>
23. (80)	6x = 4248	24. 1)163	25. (76, 80)	5)\$49.00

***26.** This table represents the equation y = 2x + 3 and shows the values of *y* when *x* is 2 and when *x* is 3. What is y when x is 4?

- 27. How many one-foot-square tiles are needed to cover the
- y = 2x + 3X У

2 3

4

7

9

?

(Inv. 3, 85) floor of a room that is 15 feet long and 10 feet wide?

*28. Find the median and mode of this set of numbers:

*29. What geometric shape is a globe? (98)

***30. a.** What is the geometric name for this solid? (63.98)

- b. How many faces does this solid have?
- **c.** Describe the angles.





Constructing Prisms

Power Ud facts mental

Power Up J

math

Find each product by the "double and half" method in **a-c.**

- a. Number Sense: 3×18
- **b.** Number Sense: 15×60
- c. Number Sense: 50×48
- d. Money: Shawntay had \$5.00. He spent \$1.75 on a birthday card for his brother. How much does he have left?
- e. Fractional Parts: What is $\frac{1}{5}$ of 100?
- **f. Estimation:** Brittany used $11\frac{3}{4}$ inches of tape to wrap one gift. About how much tape will she need to wrap five more gifts that are the same size as the first?
- g. Calculation: $\sqrt{25}$, $\div 5$, + 6, $\times 2$, 11
- h. Roman Numerals: Write XXVII in our number system.

problem Choose an appropriate problem-solving strategy to solve this solving problem. Danae can walk twice as fast as she can swim. She can run twice as fast as she can walk. She can ride a bike twice as fast as she can run. If Danae can ride her bike a guarter mile in one minute, how long would it take her to swim a guarter mile?

New Concept

In Lesson 98 we named solids by their shapes. In this lesson we will focus our attention on understanding rectangular prisms and triangular prisms.

Consider the shape of a cereal box. The shape is called a rectangular solid (or rectangular prism). Every panel, or side, of a closed cereal box is a rectangle.

If an empty cereal box or similar container is available, you may refer to it to answer the following questions:

- 1. A closed cereal box has how many panels?
- 2. What words could we use to refer to these panels?
- **3.** Without a mirror, what is the largest number of panels that can be seen at one time?
- **4.** Two panels meet at a fold, or seam, in the cardboard. Each fold is an edge. A closed cereal box has how many edges?
- **5.** Three edges meet at each corner of the box. A closed cereal box has how many corners (vertices)?

If we tape an empty cereal box closed and cut it along seven edges, we can "flatten out" the container, as shown below.



We can see the six rectangles that formed the panels of the closed box. We will use nets like this one to construct the models of solids.



Constructing Prisms

Materials needed:

- Lesson Activities 43, 44, and 45
- scissors
- tape or glue

We can make models of cubes, rectangular prisms, and triangular prisms by cutting, folding, and taping nets of shapes to form 3-dimensional figures. Use **Lesson Activities 43, 44,** and **45** to construct two rectangular prisms, two triangular prisms, and two cubes. Then study those figures to answer the questions in the Lesson Practice.

A **net** is a 2-dimensional representation of a 3-dimensional geometric figure.

Math Language

Refer to the cube to answer problems a-e.



- a. What is the shape of each face?
- b. Is each face parallel to an opposite face?
- c. Is each edge parallel to at least one other edge?
- d. Is each edge perpendicular to at least one other edge?
- e. What type of angle is formed by every pair of intersecting edges?

Refer to the rectangular prism below to answer problems f-j.



- f. What is the shape of each face?
- g. Is each face parallel to the opposite face?
- h. Is each edge parallel to at least one other edge?
- i. Is each edge perpendicular to at least one other edge?
- **j.** What type of angle is formed by every pair of intersecting edges?

Refer to the triangular prism with two faces that are equilateral triangles to answer problems **k–o**.



- k. What are the shapes of the five faces?
- I. Are the triangular faces parallel? Are the rectangular faces parallel?
- **m.** Are the triangular faces congruent? Are the rectangular faces congruent?

Lesson Practice

- n. Do you find pairs of edges that are parallel? That are perpendicular? That intersect but are not perpendicular?
- o. What types of angles are formed by the intersecting edges?

Refer to the triangular prism with two faces that are right triangles to answer problems **p–t**.



- p. What are the shapes of the five faces?
- q. Which faces are parallel?
- **r.** Are the triangular faces congruent? Are the rectangular faces congruent?
- **s.** Are there pairs of edges that are parallel? Perpendicular? Intersecting but not perpendicular?
- t. What types of angles are formed by the intersecting edges?

Written Practice

Distributed and Integrated

- *1. Fifty-three family photographs are being arranged in a photo album.
 (88) The album has 12 pages altogether, and 6 photographs can be placed on each page.
 - a. How many full pages of photographs will be in the album?
 - b. How many photographs will be on the page that is not full?
 - c. How many pages in the album will be empty?
- **2. Estimate** Abraham Lincoln was born in 1809. How old was he when ⁽⁵⁴⁾ he issued the Emancipation Proclamation in 1863?
- *3. Analyze The parking lot charges \$1.25 to park a car for the first hour. ⁽⁹⁴⁾ It charges 75¢ for each additional hour. How much does it cost to park a car in the lot for 3 hours?

- *4. **Represent** Two thirds of the team's 45 points were scored in the second half. How many points did the team score in the second half? Draw a picture to illustrate the problem.
- ***5.** Something is wrong with the sign at right. Draw two different signs that show how to correct the error.



- **6. Analyze** What is the value of 3 \$10 bills, 4 \$1 bills, 5 dimes, and 2 pennies?
- 7. **Represent** Use words to write 6412.5.
 - **8.** (59) **Estimate** Last year 5139 people attended an outdoor jazz festival. This year 6902 people attended the festival. Estimate the total attendance during those years and explain why your estimate is reasonable.
- **9. a.** Cooper opened a 1-gallon bottle of milk and poured out 1 quart. Inv. 5) How many quarts of milk were left in the bottle?
 - b. What percent of the milk was left in the bottle?
- **10.** Look at the coins below. List all of the different amounts you could make using exactly two coins.



- * **11.** Estimate the product of 39 and 41. Then find the exact product.
- **12.** Felicia slowly gave the doorknob a quarter turn counterclockwise. How many degrees did she turn the doorknob?

14. (43, 51)	\$68.57 + \$36.49	15. \$100.00 (52) <u>- \$ 5.43</u>	1 ()	6. 15 ¹⁷⁾ 24 36
17. (87)	12 <u>× 12</u>	18. \$5.08 ⁽⁵⁸⁾ <u>× 7</u>		75 21 8
19. (62, 86)	50 ²	20. $\sqrt{144}$		+ 420
21. (45, 50)	12.08 - (9.61 - 2.4)			
22. (90)	49 × 51		23. 33 × 25	
24. (80)	<u>848</u> 8		25. 9w = 6300	
* 26. (89)	Represent Draw and s	hade circles to s	how that $2\frac{2}{3}$ equa	als $\frac{8}{3}$.

*13. Five full buses held 240 students. What was the average number of

⁽⁹⁶⁾ students per bus?

***27. Represent** Draw a rectangle that is three inches long and one inch (21, Inv. 3) wide. Then find the perimeter and the area.

*28. This table represents the equation y = 3x + 1 and shows the values of *y* when *x* is 3 and when *x* is 4. What is *y* when *x* is 5?

y = 3x + 1			
x	У		
3	10		
4	13		
5	?		

* 29.	Classify Refer to this triangular prism for parts a and b.
20, 33)	a. Describe the angles as acute, right, or obtuse.

b. Which faces are parallel?



*30. This pyramid has a square base. How many vertices does the pyramid have?



Constructing Pyramids

Power Up facts

Power Up J

mental math Find each product by the "double and half" method in a-c.

- a. Number Sense: 4×14
- **b. Number Sense:** 25×80
- c. Number Sense: 50×64
- **d.** Money: Houston paid for a lawn sprinkler that cost \$8.16 with a \$10 bill. How much change should he receive?
- e. Geometry: What is the diameter of a wheel that has a radius of 14 inches?
- **f. Estimation:** Estimate 19×41 by rounding each number to the nearest ten before multiplying.
- g. Calculation: 15 9, square the number, $\div 4$, -8
- h. Roman Numerals: Write 19 in Roman numerals.

problemFranklin's family is moving to a new
house, and they have packed their
belongings in identical boxes. The
picture at right represents the stack of
boxes that is inside the moving truck.
How many boxes are in the stack?



Focus Strategy: Make It Simpler

Understand We are shown a picture of identical, stacked boxes. We assume that boxes in the upper layer are supported by boxes in the lower layers. We are asked to find how many boxes are in the stack altogether.

Plan In the picture, we can see three layers of boxes. If we can find how many boxes are in each layer, we can multiply by 3 to find the total number of boxes.

Solve If we look at the top layer of boxes, we see 4 boxes along the front and 3 boxes along the side. Four rows of 3 boxes means there are 4×3 boxes = 12 boxes in the top layer. The middle and bottom layers contain the same number of boxes as the top layer. Since there are three layers of boxes, we find that there are 3×12 boxes = **36 boxes** in the stack altogether.

Check We know our answer is reasonable because three layers of 12 boxes each is 36 boxes altogether. If we have blocks or unit cubes, we can check our answer by modeling the problem.

New Concept

Math Language

A *plane* is an endless 2-dimensional, flat surface. Lines and plane figures are found on planes. Recall from Lesson 98 that geometric shapes such as triangles, rectangles, and circles have two dimensions—length and width—but they do not have depth. These kinds of figures occupy area, but they do not take up space. We call shapes such as these plane figures because they are confined to a plane.



Shapes that take up space, such as cubes, pyramids, and cones, are *geometric solids*. Geometric solids have three dimensions: length, width, and depth. Sometimes we simply call these shapes solids. Solids are not confined to a plane, so to draw them we try to create an optical illusion to suggest their shape.



In Lesson 99 we studied models of rectangular prisms and triangular prisms. In this lesson we will study models of pyramids.

Activity

Constructing Models of Pyramids

Materials needed:

- Lesson Activity 46
- scissors
- glue or tape

Cut out the patterns for the pyramids. The shaded parts of each pattern are tabs to help hold the figures together. Fold the paper along the edges before you glue or tape the seams. You might work with a partner as you construct the models. Refer to the models to answer the questions in the Lesson Practice.

Lesson Practice

Refer to the pyramid with a square base at right to answer problems **a**–**d**.

- **a.** How many faces does the pyramid have, and what are their shapes?
- **b.** Does the pyramid have any parallel faces?
- **c.** Does the pyramid have any parallel or perpendicular edges? Explain.
- **d.** In the pyramid above, what types of angles are formed by the intersecting edges?

Refer to the pyramid with the triangular base at right to answer problems **e-h**.

e. How many faces does the pyramid have and what are their shapes?



- f. Does the pyramid have any parallel faces?
- g. Does the pyramid have any parallel or perpendicular edges?
- **h.** In the pyramid above, what types of angles are formed by intersecting edges?

1. One hundred fifty feet equals how many yards?

Written Practice

2. Tammy gave the clerk \$6 to pay for a book. She received 64¢ in ⁽⁸³⁾ change. Tax was 38¢. What was the price of the book?

Distributed and Integrated

- **3.** DaJuan is 2 years older than Rebecca. Rebecca is twice as old as
 ⁽⁹⁴⁾ Dillon. DaJuan is 12 years old. How old is Dillon? (*Hint:* First find Rebecca's age.)
- **4.** Write each decimal as a mixed number: **a.** 3.295 **b.** 32.9 **c.** 3.09
- ***5.** a. **Represent** Three fourths of the 84 contestants guessed incorrectly. How many contestants guessed incorrectly? Draw a picture to illustrate the problem.
 - b. What percent of the contestants guessed incorrectly?
- **6.** These thermometers show the average daily minimum and maximum temperatures in North Little Rock, Arkansas, during the month of January. What is the range of the temperatures?





- **7. a.** What is the diameter of this circle?
 - b. What is the radius of this circle?



8. Represent Use words to write 8.75.

*9. (P3) Three students each made a different estimate of the quotient 2589 ÷ 9. Paulo's estimate was 30, Ka'Dentia's estimate was 300, and Carter's estimate was 3000. Which student made the best estimate? Explain your answer.

*** 10.** The first five odd counting numbers are 1, 3, 5, 7, and 9. (10, 97)

Find the mean and the median of these five numbers.

* **11.** What geometric shape is a roll of paper towels?

* 12. a. Multiple Choice Which of these polygons is a parallelogram?

b. Which polygons appear to have at least one obtuse angle?

c. Which polygon does not appear to have any perpendicular sides?

13. \$16.25 - (\$6 - 50¢)		
14. $5 \times 7 \times 9$	15. \$7.83 × 6	16. 54 $ imes$ 1000
17. 45 (∞) × 45	18. 32 (⁽⁶⁷⁾ <u>× 40</u>	19. 46 × 44
20. 6) 3625	21. 5)3000	22. $7n = 987$
23. $\frac{10^3}{\sqrt{25}}$	24. \$13.76 ÷ 8	25. $\frac{234}{4}$

*26. **Represent** Draw and shade a circle to show that $\frac{8}{8}$ equals 1.

27. The perimeter of the square at right is 40 cm. What is the *(lnv. 2, Inv. 3)* area of this square? (*Hint:* First find the length of each side.)



***28. Represent** Draw a triangle that is similar to this isosceles triangle. Then draw its line of symmetry.



*29. a. Compare: 0.25 ○ 0.250
 b. Compare: \$0.25 ○ 0.25¢

***30.** One of these nets could be cut out and folded to form a cube. The ⁽⁹⁹⁾ other will not form a cube. Which net will form a cube?





Real-World

Connection

- **a.** Use a ruler to sketch the front, top, sides, and bottom of a pyramid, cone, and cube.
- **b.** In your sketches, what do the dashed lines represent?

Focus on

Probability

Many board games involve an element of **chance**. This means that when we spin a spinner, roll number cubes, or draw a card from a shuffled deck, we cannot know the outcome (result) of the event ahead of time. However, we can often find how *likely* a particular outcome is. The degree of likelihood of an outcome is called its **probability**.

10

Here we show a spinner. The face is divided into six equal parts called **sectors.** Each sector is $\frac{1}{6}$ of the face of the spinner. Assuming the spinner is balanced and fair, then a spin of the arrow can end up with the arrow pointing in any direction. The letter that names the sector where the arrow lands is the outcome of the spin. For the questions that follow, ignore the possibility that the arrow may stop on a line.



- 1. If the arrow is spun once, what outcomes are possible?
- 2. **Explain** On which letter is the arrow most likely to stop and why?
- **3. List** Write the possible outcomes of a spin in order from least likely to most likely.
- 4. Which outcome of a spin is twice as likely as the outcome C?
- 5. **Predict** If the arrow is spun many times, then about half the outcomes are likely to be which sector?
- 6. Multiple Choice If the arrow is spun many times, then what fraction of the spins are likely to stop in sector C?
 - **A** $\frac{1}{6}$ **B** $\frac{1}{3}$ **C** $\frac{1}{2}$ **D** $\frac{5}{6}$
- **7. Multiple Choice** In 60 spins, about how many times should we expect it to stop in sector C?
 - A about 6 times B about 10 times
 - C about 20 times D about 30 times

The probability of an outcome can be expressed as a number ranging from 0 to 1. An outcome that cannot happen has a probability of 0. An outcome that is **certain** to happen has a probability of 1. An outcome that could happen but is not certain to happen is expressed as a fraction between 0 and 1.

Use the spinner at right to answer problems 8–10.

- 8. **Explain** What is the probability that the arrow will stop in sector D? Why?
- **9. Explain** What is the probability that the outcome will be one of the first three letters of the alphabet? Why?
- **10.** What is the probability that the arrow will stop in sector C?
- We show a standard dot cube at right.
 - **11.** What numbers are represented by the dots on the faces of a dot cube?
 - **12. Justify** If a dot cube is rolled once, which number is most likely to end up on top? Why?
 - **13. Multiple Choice** If a dot cube is rolled many times, about how often would we expect to roll a number greater than 3?
 - A less than half the time **B** about half the time
 - **C** more than half the time **D** none of the time
 - 14. If a dot cube is rolled once, what is the probability of rolling a 7?
 - 15. With one roll of a dot cube, what is the probability of rolling a 1?
 - **16. Multiple Choice** How would we describe the likelihood of rolling a 6 with one roll of a dot cube?
 - A very likely **B** just as likely to roll a 6 as not to roll a 6
 - C unlikely D certain

The **chance** of an event is sometimes expressed as a percent from 0% to 100%. For example, if a meteorologist forecasts that the chance of rain is 20%, then the meteorologist is stating that it might rain, but that it is more likely not to rain. A forecast of 100% chance of rain means that the meteorologist believes it is certain to rain.

- **17.** The weather forecast stated that the chance of rain is 40%. According to the forecast, is it more likely to rain or not to rain?
- **18.** The meteorologist said that the chance of rain is 80%. This means that the chance it will not rain is what percent?

Probability Experiments

Materials needed:

- dot cube
- Lesson Activity 47





Experiment 1: Work with a partner for this experiment. You and your partner will roll one dot cube 36 times and tally the number of times each face of the dot cube turns up. You will record the results in the "Experiment 1" table on **Lesson Activity 47.** (A copy of the table is shown below.)

Predict Before starting the experiment, predict the number of times each outcome will occur during the experiment. Write your predictions in the column labeled "Prediction."

Outcome	Prediction	Tally	Total Frequency	
1				
2				
3				
4				
5				
6				

36 Rolls of One Dot Cube

Now begin rolling the dot cube. Make a tally mark for each roll in the appropriate box in the "Tally" column. When all groups have finished, report your results to the class. As a class, total the groups' tallies for each outcome, and write these totals in the boxes under "Total Frequency."

19. Make a bar graph using the data from your table.

20. What conclusions can you draw from the results of Experiment 1?

21. Is it easier to compare data using the bar graph or the table?

Experiment 2: In this experiment, you and your group will roll a pair of dot cubes 36 times and tally the outcomes. For each roll, the outcome will be the sum of the two numbers that end up on top. You will record your results in the "Experiment 2" table on **Lesson Activity 47.**

Form groups so that each group can have two number cubes.

Predict Before starting the experiment, predict as a group the number of times each outcome will occur during the experiment. Write your predictions in the column labeled "Prediction."

36 Rolls of Two Dot Cubes				
Outcome	Prediction	Tally	Total Frequency	
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				

Now begin rolling the dot cubes. Each time you roll a pair of dot cubes, make a tally mark in the appropriate box. When all groups have finished, report your results to the class. As a class, total the groups' tallies for each outcome and record these totals in the "Total Frequency" column.

- 22. Which outcome(s) occurred most frequently? Why?
- 23. Which outcome(s) occurred least frequently? Why?
- 24. What conclusions can you draw from the results of Experiment 2?
- **25.** What are all the possible combinations you could roll with a sum of 7 as the result? Explain.